

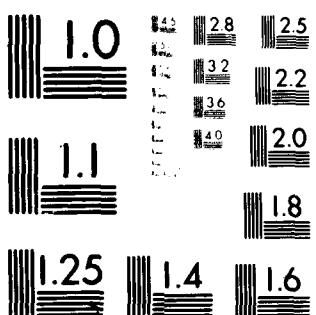
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# NONPARAMETRIC TESTS OF INDEPENDENCE FOR CENSORED DATA

by

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## AFOSR Final Technical Report

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Amherst, Massachusetts



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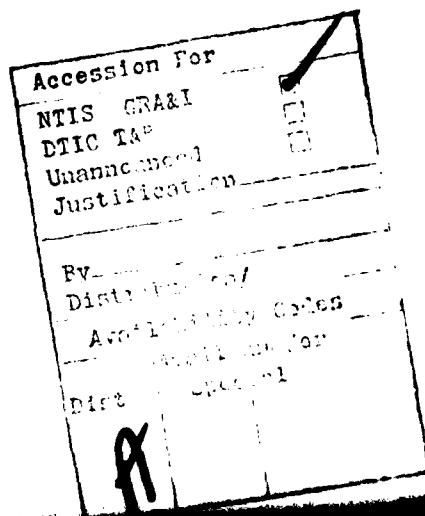
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#### ABSTRACT

The work accomplished is represented by six Tech Reports already issued. Papers based on two of them are accepted for publication and are soon to be published in two of the leading journals in statistics. One other is submitted for publication. And yet another will appear in a Proceedings of a conference on nonparametric statistics.

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A. D. ELOSE  
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## 1. Introduction.

The accomplishments are represented by the following Technical Reports (listed in chronological order) written and issued from time to time:

- [1] Korwar, R.M. (1979). On the uniformly minimum variance unbiased estimators of the variance and its reciprocal of an inverse Gaussian distribution.
- [2] Dahiya, R.C. (1979). Estimating the population sizes of different types of organisms in a plankton sample.
- [3] Dahiya, R.C. (1979). An improved method of estimating an integer parameter by maximum likelihood.
- [4] Dahiya, R.C. (1979). Pearson goodness-of-fit test when the sample size is unknown.
- [5] Korwar, R.M. (1980). Nonparametric estimation of a bivariate survivorship function with doubly censored data.
- [6] Hollander, M., and Korwar, R.M. (1980). Nonparametric Bayesian estimation of the horizontal distance between two populations.

## 2. A Brief Description of the Work by Korwar.

In [5] above, the problem of estimation of a bivariate survivorship function with doubly censored data is considered. A "self-consistent" estimator is developed by first "reducing" the problem to that of estimation in a singly and right censored situation. This estimator is shown to satisfy a likelihood equation, and its uniqueness is investigated. The results obtained naturally parallel those obtained by Campbell (1979) in the singly censored case.

This paper is submitted for publication to the Annals of Statistics. A revision is underway. An abstract of the paper has appeared in the Bulletin of the Institute of Mathematical Statistics (IMS Bulletin 9, #3 (1980), 131, #80t-39).

In [1] above, it is argued that the two-parameter inverse Gaussian distribution has useful applications in a wide variety of fields such as reliability and biometry. The uniformly minimum variance unbiased estimator of the mean of the distribution is known and is the sample mean. However, no such estimator of the variance is reported in the literature. Here the uniformly minimum variance unbiased estimators of the variance and its

reciprocal are derived.

A revised version of the Tech Report is accepted for publication in the Journal of the American Statistical Association and will appear in that journal in September, 1980.

In [6] above, Hollander and Korwar consider the problem of nonparametric Bayesian estimation of the horizontal distance  $\Delta(x) = G^{-1}(F(x)) - x$  between two distribution functions  $F$  and  $G$  at  $x$ . Doksum (1974) has shown  $\Delta(x)$  to be a useful measure of the difference, at each  $x$ , between the populations defined by continuous distribution functions  $F$  and  $G$ . Here the present authors assume that  $G$  is known, and develop a Bayesian nonparametric estimator  $\tilde{\Delta}_n(x)$  of  $\Delta(x)$  based on a random sample of  $n$   $X$ 's from  $F$ . The estimator  $\tilde{\Delta}_n$  is, for weighted squared-error loss, Bayes with respect to Ferguson's (1973) Dirichlet process prior. Using a result of Korwar and Hollander (1976), the Bayes risk of  $\tilde{\Delta}_n$  is evaluated for the case when  $G$  is uniform.

This is an invited paper (Hollander) and will appear in a North-Holland Publication of the Proceedings of the János Bolyai Nonparametric Conference, Budapest, Hungary, June, 1980.

In Section 2 of Korwar and Dahiya (1979) a nonparametric estimator of the bivariate survival function, when only one known variable is assumed to be censored, was proposed and studied. Korwar has now further studied this estimator for its large sample properties. From the properties of the univariate Kaplan-Meier estimator (1958) it follows that the above mentioned Korwar-Dahiya estimator is pointwise consistent. From the work of Campbell and Földes (1980) it also follows that under mild conditions the estimator obeys a law of the iterated logarithms. Details will be reported later on as part of the work under a new AFOSR Grant.

### 3. A Brief Description of the Work by Dahiya.

In [2] above, Dahiya considers a problem related to the estimation of the binomial parameter  $N$ . Biologists are interested in estimating the population sizes of different types of organisms in a plankton sample by making use of their count in the subsamples. A method is presented for determining subsample sizes for the different types of organisms in the

original sample. Also, the maximum likelihood estimators of the population sizes and their asymptotic properties are examined. The author also proposes a test for investigating the clustering of the organisms and the interval estimation problem of the organism count is examined for this case. Finally, the author demonstrates the use of the results obtained here using some plankton samples taken from the Chesapeake Bay.

This paper is accepted for publication and will appear in Biometrics in September, 1980.

In [3] above, a simple graphical method is described for obtaining the maximum likelihood of an integer-valued parameter. The method does not use calculus and is easy to comprehend. The use of this method is shown in the specific cases of the binomial, Poisson, and the exponential distributions. A numerical example is also provided for the comparision of this method with the method used previously.

In [4] above, Dahiya considers the estimation of the sample size  $N$  from truncated samples and the use of the Pearson chi-square goodness-of-fit test statistic for the situation. In order to use the Pearson chi-square goodness-of-fit test statistic for testing the adequacy of the assumed model, one needs to examine the effect of the estimation of  $N$  on the distribution of this test statistic. Here, it is shown that, when  $N$  itself is estimated, the asymptotic distribution of this test statistic is the same as the weighted sum of two independent chi-square random variables. Furthermore, the asymptotic distribution of this statistic for specific models is considered. An example is given to illustrate the use of the results obtained here.

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